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1. Express

$$\frac{3x^2}{(2x^2+7x+6)} \times \frac{7(3+2x)}{3x^5}$$

as a single fraction in its simplest form.

(4)

$\times$	$2x$	$+3$
x	$2x^2$	$3x$
$+2$	$4x$	6

Factorise denominator

$$\frac{3x^2 \times 7(3+2x)}{(x+2)(2x+3) \times 3x^5}$$

Place in one fraction

$$\frac{3(7)x^2\,(2x+3)}{3\,x^5\,(x+2)(2x+3)}$$

Rearrange

$$\frac{\cancel{3}(7)\,\cancel{x^2}\,\cancel{(2x+3)}}{\cancel{3}\,x^{\cancel{5}\,3}(x+2)\cancel{(2x+3)}}$$

cancle terms

$$\boxed{\frac{7}{x^3(x+2)}}$$

answer

Leave blank

2. The function f is defined by

$$f : x \mapsto 2x, \quad x \in \mathbb{R}.$$

(a) Find $f^{-1}(x)$ and state the domain of f^{-1}.

(2)

The function g is defined by

$$g : x \mapsto 3x^2 + 2, \quad x \in \mathbb{R}.$$

(b) Find $gf^{-1}(x)$.

(2)

(c) State the range of $gf^{-1}(x)$.

(1)

a) $f(x) = 2x$ this means x is a real number

Domain of $f(x)$ $\boxed{x \in \mathbb{R}}$

$x \in \mathbb{C}$	complex	
$x \in \mathbb{R}$	Real	
$x \in \mathbb{Q}$	Rational	
$x \in \mathbb{Z}$	Integer	
$x \in \mathbb{Z}^+$	Positive Integer	
$x \in \mathbb{N}$	Natural	

$\therefore$ Range of $f(x)$ $f(x) \in \mathbb{R}$

Range of $f(x)$: Domain of $f^{-1}(x)$

$\therefore$ $\boxed{x \in \mathbb{R}}$

$f(x) : 2x$

$x = \frac{1}{2} f(x)$ Rearrange to make x the subject

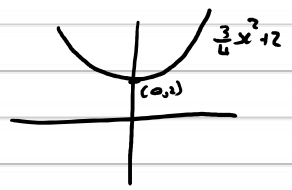
$\therefore$ $\boxed{f^{-1}(x) = \frac{1}{2}x}$ swap terms

b) $g(x) = 3x^2 + 2$, $f^{-1}(x) = \frac{1}{2}x$

$g[f^{-1}(x)] = 3(\frac{1}{2}x)^2 + 2$

$\boxed{= \frac{3}{4}x^2 + 2}$

c)



from graph

Range : $\boxed{gf^{-1}(x) \geq 2}$

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3. Find the exact solutions of

(i) $e^{2x+3} = 6$,

(3)

(ii) $\ln(3x + 2) = 4$.

(3)

(i) $\ln(e^{2x+3}) = \ln(6)$ | $\ln(e^a) = a$

$2x+3 = \ln(6)$ | $e^{\ln(a)} = a$

$2x = \ln(6) - 3$

$\boxed{x = \frac{1}{2}[\ln(6) - 3]}$

(ii) $\ln(3x+2) = 4$

$e^{\ln(3x+2)} = e^4$

$3x+2 = e^4$

$3x = e^4 - 2$

$\boxed{x = \frac{1}{3}[e^4 - 2]}$

Leave blank

4. Differentiate with respect to x

(i) $x^3 e^{3x}$, **(3)**

(ii) $\dfrac{2x}{\cos x}$, **(3)**

(iii) $\tan^2 x$. **(2)**

Given that $x = \cos y^2$,

(iv) find $\dfrac{dy}{dx}$ in terms of y. **(4)**

i) $\dfrac{d}{dx} x^3 e^{3x}$ | Using the Product rule

$x^3 \dfrac{d}{dx} e^{3x} + e^{3x} \dfrac{d}{dx} x^3$ | $\dfrac{d}{dx} uv = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$x^3 (3e^{3x}) + e^{3x}(3x^2)$

$\boxed{3x^2 e^{3x}(x+1)}$

ii) $\dfrac{2x}{\cos(x)} = 2x\sec(x)$ | Easier form [Quotient rule is also acceptable]

$\dfrac{d}{dx} 2x\sec(x)$ | Product rule

$2x \dfrac{d}{dx}\sec(x) + \sec(x)\dfrac{d}{dx}(2x)$ | $\dfrac{d}{dx} uv = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$2x\sec(x)\tan(x) + \sec(x)(2)$

$\boxed{2\sec(x)[x\tan(x) + 1]}$

Leave blank

4. Differentiate with respect to x

~~(i) $x^3 e^{3x}$,~~ **(3)**

~~(ii) $\frac{2x}{\cos x}$,~~ **(3)**

(iii) $\tan^2 x$. **(2)**

Given that $x = \cos y^2$,

(iv) find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of y. **(4)**

(iii)

$\frac{d}{dx} \tan^2(x)$

chain Rule

$\frac{dy}{dx} = \frac{dy}{\cancel{du}} \frac{\cancel{du}}{dx}$

$u = \tan(x)$

$\frac{du^2}{du} \times \frac{du}{dx}$

$2u \times \sec^2(x)$

$\boxed{2\tan(x)\sec^2(x)}$

iv)

$x = \cos(y^2)$

$u = y^2$

chain Rule

$\frac{d}{du}\cos(u) \times \frac{d}{dy} u$

$\frac{dx}{dy} = \frac{dx}{du} \times \frac{du}{dy}$

$\frac{dx}{dy} = -\sin(y^2) \times 2y$

$\frac{dy}{dx} = \left(\frac{dx}{dy}\right)^{-1}$

$\therefore \frac{dy}{dx} = \boxed{\frac{-1}{2y}\operatorname{cosec}(y^2)}$

Leave blank

5. (a) Using the formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B,$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B,$$

show that

(i) $\sin (A + B) - \sin (A - B) = 2 \cos A \sin B,$

(2)

(ii) $\cos (A - B) - \cos (A + B) = 2 \sin A \sin B.$

(2)

(b) Use the above results to show that

$$\frac{\sin(A + B) - \sin(A - B)}{\cos(A - B) - \cos(A + B)} = \cot A.$$

(3)

Using the result of part (b) and the exact values of sin 60° and cos 60°,

(c) find an exact value for cot 75° in its simplest form.

(4)

a) $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

i) $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$

$\therefore \sin(A+B) - \sin(A-B) = \cancel{\sin(A)\cos(B)} + \cos(A)\sin(B) - \cancel{\sin(A)\cos(B)} - -\cos(A)\sin(B)$

$\boxed{= 2\cos(A)\sin(B)}$

ii) $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$\therefore \cos(A-B) - \cos(A+B) = \cancel{\cos(A)\cos(B)} + \sin(A)\sin(B) - \cancel{\cos(A)\cos(B)} - -\sin(A)\sin(B)$

$\boxed{= 2\sin(A)\sin(B)}$

b) $\dfrac{\cancel{2}\cos(A)\cancel{\sin(B)}}{\cancel{2}\sin(A)\cancel{\sin(B)}} = \dfrac{\cos(B)}{\sin(B)}$

$\boxed{= \cot(A)}$

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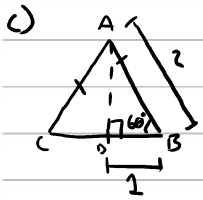
5. continued

c) 

ABC is an equilateral triangle

$\therefore \angle ABC = 60^\circ$

D is the midpoint of BC

$$|AD| = \sqrt{2^2 - 1^2}$$
$$= \sqrt{3}$$

$\sin(\alpha) = \frac{o}{h}$, $\cos(\alpha) = \frac{a}{h}$ $\quad \therefore \sin(60) = \frac{\sqrt{3}}{2}$, $\cos(60) = \frac{1}{2}$

Find $\cot(75^\circ)$

we know $\frac{\sin(A+B) - \sin(A-B)}{\cos(A-B) - \cos(A+B)} = \cot(A)$

so let $A = 75^\circ$

We need $A - B = 60^\circ$

$\therefore$ Let $B = 15^\circ$

$$\frac{\sin(75+15) - \sin(75-15)}{\cos(75-15) + \cos(75+15)} = \frac{\sin(90) - \sin(60)}{\cos(60) - \cos(90)}$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2} - 0}$$

$$\boxed{= 2 - \sqrt{3}}$$

Leave blank

6.

Figure 1

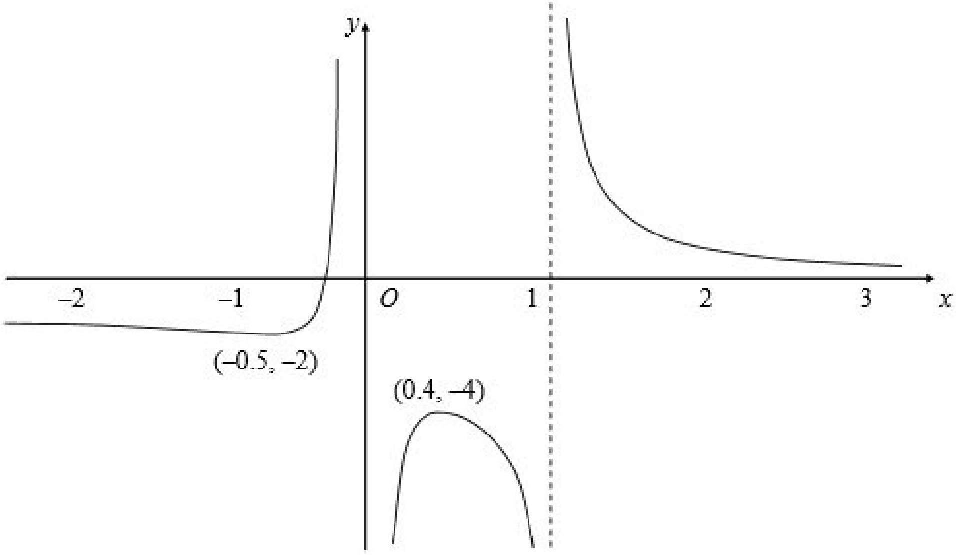


Figure1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve has a minimum point at (–0.5, –2) and a maximum point at (0.4, –4). The lines $x = 1$, the x-axis and the y-axis are asymptotes of the curve, as shown in Fig. 1.

On a separate diagram sketch the graphs of

(a) $y = |\mathrm{f}(x)|$,
(4)

(b) $y = \mathrm{f}(x - 3)$,
(4)

(c) $y = \mathrm{f}(|x|)$.
(4)

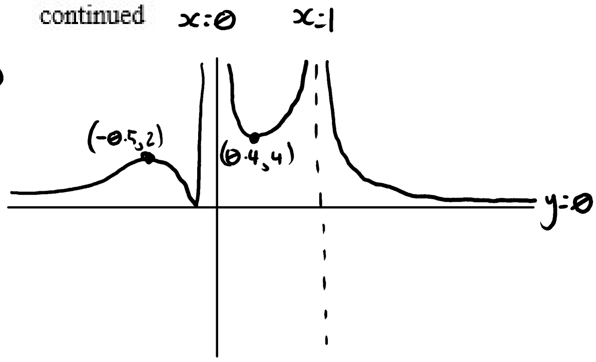
In each case show clearly

(i) the coordinates of any points at which the curve has a maximum or minimum point,

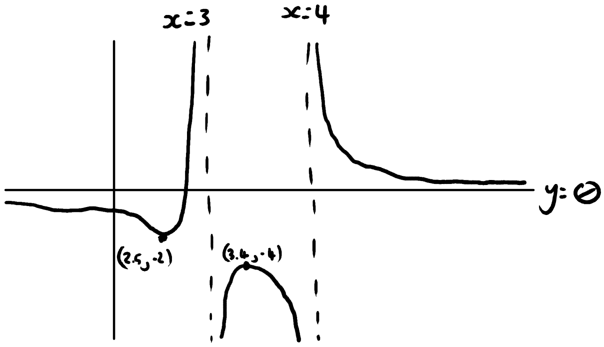
(ii) how the curve approaches the asymptotes of the curve.

6. continued

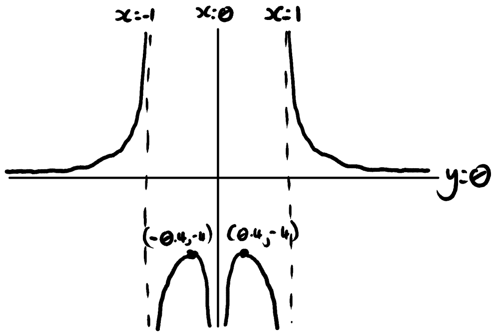
a)



b)



c)



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7. (a) Sketch the curve with equation $y = \ln x$.

(2)

(b) Show that the tangent to the curve with equation $y = \ln x$ at the point (e, 1) passes through the origin.

(3)

(c) Use your sketch to explain why the line $y = mx$ cuts the curve $y = \ln x$ between $x = 1$ and $x = e$ if $0 < m < \frac{1}{e}$.

(2)

Taking $x_0 = 1.86$ and using the iteration $x_{n+1} = e^{\frac{1}{3}x_n}$,

(d) calculate x_1, x_2, x_3, x_4 and x_5, giving your answer to x_5 to 3 decimal places.

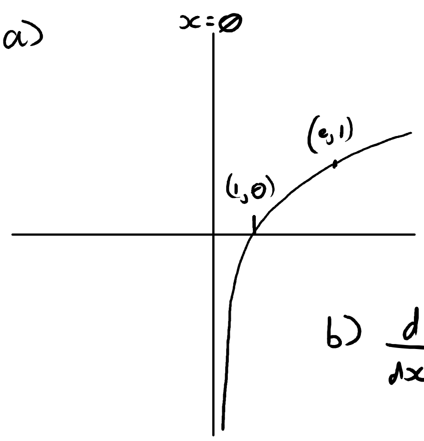
(3)

The root of $\ln x - \frac{1}{3}x = 0$ is α.

(e) By considering the change of sign of $\ln x - \frac{1}{3}x$ over a suitable interval, show that your answer for x_5 is an accurate estimate of α, correct to 3 decimal places.

(3)

a)



b) $\frac{d}{dx}\ln(x) = \frac{1}{x}$ | gradient of curve

$y - 1 = \frac{1}{e}(x - e)$ | Find equation of the tangent

$y - 1 = \frac{1}{e}x - 1$

$y = \frac{1}{e}x$ | $y = mx + c$, $c = 0$, y intercept $= 0$ ∴ passes origin

(0, 0) is a valid solution to this equation

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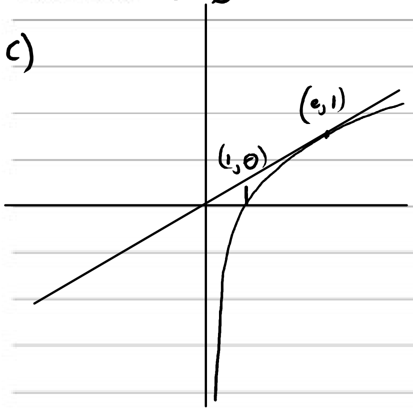
7. continued

c)



all lines $y=mx$ pass through $(0,0)$

gradient $m>0$ means that y is positive always in domain of $\ln(x)$ $[x<0]$

gradient $m<\frac{1}{e}$ means line $y=mx$ will always be below line $y=\frac{1}{e}x$

x values of intercepts at extreams

$y=0x$, $y=\ln(x)$	$y=\frac{1}{e}x$, $y=\ln x$	$\therefore$ x value of intercept
$0=\ln(x)$	$\frac{1}{e}x=\ln x$	$1<x<e$
$x=1$	$x=e$	

d) $x_{n+1} = e^{\frac{1}{3}x_n}$

n	x_n
0	1.860
1	1.859
2	1.858
3	1.858
4	1.858
5	1.857

in your calculator type:

1.86, hit equals

then

$e^{\frac{Ans}{3}}$, hit equals for x_1

again for x_2

etc.

e) $[1.8565, 1.8575]$

All values $1.8565 \leqslant n < 1.8575$ round to 1.857

$f(x)=\ln(x)-\frac{1}{3}x$

$f(1.8565)=-1.40\times10^{-4}$

$f(1.8575)=6.48\times10^{-5}$

Statement Required

Change in sign of a continuous function in given region, Therefore root present in given region.

Leave blank

8. In a particular circuit the current, I amperes, is given by

$$I = 4\sin\theta - 3\cos\theta, \quad \theta > 0,$$

where θ is an angle related to the voltage.

Given that $I = R\sin(\theta - \alpha)$, where $R > 0$ and $0 \leqslant \alpha < 360°$,

(a) find the value of R, and the value of α to 1 decimal place.
(4)

(b) Hence solve the equation $4\sin\theta - 3\cos\theta = 3$ to find the values of θ between 0 and 360°.
(5)

(c) Write down the greatest value for I.
(1)

(d) Find the value of θ between 0 and 360° at which the greatest value of I occurs.
(2)

$R\sin(\theta-\alpha) = R\cos(\alpha)\sin(\theta) - R\sin(\alpha)\cos(\theta)$

$R\cos(\alpha) = 4$

$R\sin(\alpha) = 3$

$[R\cos(\alpha)]^2 = 16$

$[R\sin(\alpha)]^2 = 9$

$\dfrac{R\sin(\alpha)}{R\cos(\alpha)} = \tan(\alpha)$

$= \dfrac{3}{4}$

$R^2\cos^2(\alpha) + R^2\sin^2(\alpha) = 16+9$

$R^2(\cos^2(\alpha)+\sin^2(\alpha)) = 25$

$\tan^{-1}(\frac{3}{4}) = 36.9$ (1dp)

$\boxed{\alpha = 36.9}$ (1dp)

$R^2 \times 1 = 25$

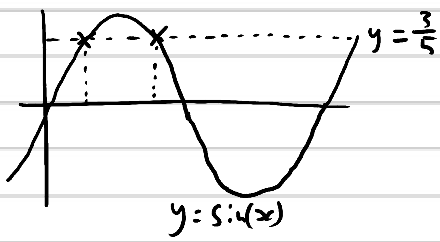
$\boxed{R = 5}$

b) $5\sin(\theta - 36.9) = 3$

$\sin(\theta - 36.9) = \frac{3}{5}$

$\theta - 36.9 = 36.9, 143$

$\boxed{\begin{aligned}\theta &= 73.74° \\ &= 180°\end{aligned}}$



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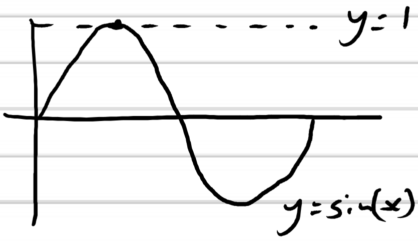
8. continued

c) maximum value of $5\sin(\theta - \alpha)$ is $\underline{5}$

d) $5\sin(\theta - 36.9) = 5$

$\sin(\theta - 36.9) = 1$



$\theta - 36.9 = 90°$

$\theta = 36.9 + 90$

$\boxed{\theta = 126.9°}$

no other values in domain

END